III Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (N.S.) (2012-13 and Onwards)
MATHEMATICS – III

Time : 3 Hours   Max. Marks : 100

Instruction : *Answer* ***all*** *questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Prove that every subgroup of an abelian group is normal.

2) Is f : (z, +) → (2z, +) defined by f(x) = 2x a homomorphism.

3) Define Kernel of a homomorphism.

4) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$. Find $f^{-1} \circ g$.

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5) Solve $x - 2y \geq 1$ graphically.

6) Define : (1) Basic solution (2) Basic feasible solution of a L.P.P.

7) Determine the initial basic feasible solution to the following transportation problem using Row minima method.

Origin \ Destination	D_1	D_2	D_3	Supply
O_1	50	30	220	1
O_2	90	45	170	3
O_3	200	250	50	4
Demand	4	2	2	

8) Find the limit of the sequence $\left\{\frac{1}{2^n}\right\}$.

9) Show that the sequence $\{x_n\}$ whose n^{th} term is $\frac{1}{3n+4}$ is monotonically decreasing.

10) Define limit of a sequence.

11) State Cauchy's principle of convergence of sequence.

12) If a series ΣU_n is convergent, then prove that $\lim_{n \to \infty} u_n = 0$.

13) Test the convergence of the series $\sum \frac{n^3}{1+n^4}$.

14) State D'Alembert's ratio test for infinite series.

15) Define conditional convergence and absolute convergence of an alternating series.

16) Prove that $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

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17) Verify Rolle's theorem for the function defined by $f(x) = x^2 - 2x$ in [0, 2].

18) State Lagrange's mean value theorem.

19) Expand $\log_e(1 + x)$ upto the term containing x^2 by Maclaurin's expansion.

20) Evaluate $\lim_{x \to 0}\left(\frac{1-\cos x}{x^2}\right)$.

II. Answer **any two** questions. **(2×5=10)**

1) Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H \; \forall g \in G$.

2) If $f : G \to G'$ is a homomorphism from the group G into the group G′ then prove that (1) $f(e) = e'$ where e and e′ are the identity elements of G and G′ respectively. (2) $f(a^{-1}) = [f(a)]^{-1} \; \forall \; a \in G$.

3) If $f : G \to G'$ is a homomorphism from the group G into the group G′ with Kernel K, then prove that K is a normal subgroup of G.

4) If $G = \{a + b\sqrt{2} \mid a, b \in Q\}$ show that $f : (G, +) \to (G, +)$ defined by $f(a + b\sqrt{2}) = a - b\sqrt{2}$ is an isomorphism.

(3×5=15)

III. Answer **any three** questions :

1) Solve LPP graphically :

Maximize $z = 3x + 5y$ subject to the constraints $x + 2y \leq 20$, $x + y \leq 15$, $y \leq 6$
$x, y \geq 0$.

2) Find all the basic feasible solution of the equation $2x_1 + 6x_2 + 2x_3 + x_4 = 3$,
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$.

3) Solve by simplex method.

Maximize $Z = 6x_1 + 2x_2 + 3x_3$ subject to the constraints $6x_1 + 5x_2 + 3x_3 \leq 26$,
$4x_1 + 2x_2 + 5x_3 \leq 7$, $x_1, x_2, x_3 \geq 0$.

4) Obtain an initial solution of the following transportation problem using Vogel's approximation method.

Warehouse / Godown	W_1	W_2	W_3	Availability
G_1	5	1	3	34
G_2	6	5	2	15
G_3	4	3	1	12
G_4	3	4	6	19
Requirement	25	31	24	

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(2×5=10)

IV. Answer **any two** questions :

1) If $\lim_{n\to\infty} a_n = l$ and $\lim_{n\to\infty} b_n = m$ then prove that $\lim_{n\to\infty} (a_n . b_n) = lm$.

2) Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{n+n}$ is monotonically increasing and bounded.

3) Test the convergence of the sequences whose n^{th} terms are (1) $\frac{\log n}{n}$

(2) $\frac{(3n+1)(n+2)}{n(n-1)}$.

V. Answer **any four** questions.

(4×5=20)

1) Discuss the convergence of the series $\sum \frac{(n^2+n+1)}{(n^4+1)}$.

2) State and prove Cauchy's root test for a series of positive terms.

3) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{2.4.6...(2n)}.x^n$.

4) Examine the convergence of $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n(n-1)}$, $0 < x < 1$.

5) Sum to infinity the series.

$$1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$$

6) Sum to infinity the series.

$$1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \dots$$

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VI. Answer **any three** questions.

(3×5=15)

1) Examine the differentiability of the function

$$f(x) = \begin{cases} 1-3x & \text{for } 0 < x \leq 1 \\ x^2 - 5x + 2 & \text{for } 1 < x \leq 2 \end{cases} \text{ at } x = 1.$$

2) Show that a function which is continuous in a closed interval attain its bounds atleast once in the interval.

3) State and prove Cauchy's mean value theorem.

4) Obtain Maclaurin's expansion of $\log_e(\sec x)$ upto the term containing x^3.

5) Evaluate $\lim_{x \to 0} (\cos x)^{1/x^2}$.
